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SOME NEW GENERAL CONGRUENCE PROPERTIES FOR $p_r(n)$

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Abstract. We study certain supplementary congruences for $p_r(n)$ where n and r are non-negative integers with the application of theta function identities which are attributed to Ramanujan.

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1. Introduction

Throughout the paper, we assume that $|q| < 1$ and employ the standard notation

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k).$$

A principal case of $f(a,b)$ is the Euler's pentagonal number theorem,

$$f(-q) := f(-q,-q^2) = \sum_{k=-\infty}^{\infty}(-1)^k q^{k(3k-1)/2} = (q;q)_\infty.$$

For convenience, we write $f_n := f(-q^n)$.
In 1918, Ramanujan [5, p. 192-193] set forth the discussion of the general partition function for any non-negative and non-zero integer represented as n and r, and denoted by $p_r(n)$ as

$$\sum_{n=0}^{\infty} p_r(n)q^n = \frac{1}{(q;q)_\infty^r},\ |q|<1. \tag{1}$$

For $r = 1$, $p_1(n)$ represents the partition function which counts the number of unrestricted partition of any given non-negative integer n. For rotational convenience $p_1(n)$ can be denoted as $p(n)$. Ramanujan [5] asserted that a positive integer λ and any prime $\overline{\nu}$, which can be represented as $6\lambda - 1$, satisfies

$$p_{-4}\left(n\overline{\nu} - \frac{\overline{\nu}+1}{6}\right) \equiv 0 \pmod{\overline{\nu}}.$$

The congruence properties of the partition function $p_r(n)$ for certain negative value of r have studied by Ramanathan [13], Atkin [1], Ono [11], Saikia and Chetry [14], Srivatsava et al., [16], Murugan and Fathima [9]. Also the congruence properties of the partition function $p_r(n)$ for certain positive values of r have been studied by Gandhi [7], Newman [10], Baruah and Ojah [3]. Recently, Saikia and Baruah [15] discussed certain new properties of the

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general partition function $p_r(n)$ modulo 5 and 7 for certain positive values of r. Also Srivasta et al., [12] have established certain general congruence properties of $p_r(n)$ modulo 13 for some positive values of r.
In this paper, we study infinite family of congruences for $p_r(n)$ modulo 13, where r is positive. For non-negative integer k we establish some infinite family of congruences for $p_r(n)$ modulo p^2, where r is $p^2k + p^2 - 1$ and $p^2k + p^2 - 3$. The following are our main results.

Theorem 1.1. *Let k be any non-negative integer, we have*

$$p_{13k+10}(13n + \nu) \equiv 0 \pmod{13}, \tag{2}$$

$$p_{13k+12}(13n + \mu) \equiv 0 \pmod{13}, \tag{3}$$

where $\nu \in \{4, 5, 7, 8, 9, 11, 12\}$ and $\mu \in \{3, 4, 6, 8, 10, 11\}$.

Theorem 1.2. *Let k be any non-negative integer, we have*

$$p_{169k+8}(169n + 13\nu + 9) \equiv 0 \pmod{13}, \tag{4}$$

$$p_{169k+9}(169n + 13\nu + 2) \equiv 0 \pmod{13}, \tag{5}$$

$$p_{169k+32}(169n + 13\nu + 10) \equiv 0 \pmod{13}, \tag{6}$$

$$p_{169k+33}(169n + 13\nu + 3) \equiv 0 \pmod{13}, \tag{7}$$

$$p_{169k+56}(169n + 13\nu + 11) \equiv 0 \pmod{13}, \tag{8}$$

$$p_{169k+57}(169n + 13\nu + 4) \equiv 0 \pmod{13}, \tag{9}$$

$$p_{169k+80}(169n + 13\nu + 12) \equiv 0 \pmod{13}, \tag{10}$$

$$p_{169k+81}(169n + 13\nu + 5) \equiv 0 \pmod{13}, \tag{11}$$

$$p_{169k+105}(169n + 13\nu + 6) \equiv 0 \pmod{13}, \tag{12}$$

$$p_{169k+129}(169n + 13\nu + 7) \equiv 0 \pmod{13}, \tag{13}$$

where $\nu \in \{4, 5, 7, 8, 9, 11, 12\}$.

Theorem 1.3. *Let k be any non-negative integer, we have*

$$p_{169k+34}(169n + 13\mu + 9) \equiv 0 \pmod{13}, \tag{14}$$

$$p_{169k+35}(169n + 13\mu + 2) \equiv 0 \pmod{13}, \tag{15}$$

$$p_{169k+58}(169n + 13\mu + 10) \equiv 0 \pmod{13}, \tag{16}$$

$$p_{169k+59}(169n + 13\mu + 3) \equiv 0 \pmod{13}, \tag{17}$$

$$p_{169k+82}(169n + 13\mu + 11) \equiv 0 \pmod{13}, \tag{18}$$

$$p_{169k+83}(169n + 13\mu + 4) \equiv 0 \pmod{13}, \tag{19}$$

$$p_{169k+106}(169n + 13\mu + 12) \equiv 0 \pmod{13}, \tag{20}$$

$$p_{169k+107}(169n + 13\mu + 5) \equiv 0 \pmod{13}, \tag{21}$$

$$p_{169k+131}(169n + 13\mu + 6) \equiv 0 \pmod{13}, \tag{22}$$

where $\mu \in \{3,4,6,8,10,11\}$.

Theorem 1.4. *Let k be any non-negative integer, we have*

$$p_{169k+47}(169n+13\kappa+9) \equiv 0 \pmod{13}, \tag{23}$$

$$p_{169k+48}(169n+13\kappa+2) \equiv 0 \pmod{13}, \tag{24}$$

$$p_{169k+71}(169n+13\kappa+10) \equiv 0 \pmod{13}, \tag{25}$$

$$p_{169k+72}(169n+13\kappa+3) \equiv 0 \pmod{13}, \tag{26}$$

$$p_{169k+95}(169n+13\kappa+11) \equiv 0 \pmod{13}, \tag{27}$$

$$p_{169k+96}(169n+13\kappa+4) \equiv 0 \pmod{13}, \tag{28}$$

$$p_{169k+119}(169n+13\kappa+12) \equiv 0 \pmod{13}, \tag{29}$$

$$p_{169k+120}(169n+13\kappa+5) \equiv 0 \pmod{13}, \tag{30}$$

where $\kappa \in \{1,2,3,4,5,6,7,8,9,10,11,12\}$.

Theorem 1.5. *For any prime* $p \geq 5$, $k \equiv -1 \pmod{p}$ *and* $1 \leq \nu \leq p-1$, *we have*

$$p_{p^2k+p^2-1}\left(p^2n+p\nu+\frac{p^2-1}{24}\right) \equiv 0 \pmod{p^2}. \tag{31}$$

Theorem 1.6. *For any odd prime p and* $1 \leq \nu \leq p-1$, *we have*

$$p_{p^2k+p^2-3}\left(p^2n+p\nu+\frac{p^2-1}{8}\right) \equiv 0 \pmod{p^2}. \tag{32}$$

2. Preliminaries

We first collect some necessary notation and identities which are needed to prove the main results of this paper.

Ramanujan's general theta-function $f(a,b)$ [4, p.35, Entry 19] is defined by

$$f(a,b)=\sum_{n=-\infty}^{\infty} a^{n(n+1)/2}b^{n(n-1)/2}=(-a,ab)_\infty(-b,ab)_\infty(ab,ab)_\infty,\ |ab|<1$$

The following 13-dissection identity holds.

From Berndt [4, p. 373, Entry 8(i), eq. (8.1)], we have

$$\begin{aligned} f_1 = f_{169}\bigg(&\frac{L(q^{13})}{J(q^{13})}-q\frac{N(q^{13})}{K(q^{13})}-q^2\frac{J(q^{13})}{I(q^{13})}+q^5\frac{M(q^{13})}{L(q^{13})} \\ &+q^7-q^{12}\frac{K(q^{13})}{M(q^{13})}+q^{22}\frac{I(q^{13})}{N(q^{13})}\bigg), \end{aligned} \tag{33}$$

where

$$I(q)=:f(-q,-q^{12}), J(q)=:f(-q^2,-q^{11}), K(q)=:f(-q^3,-q^{10}),$$
$$L(q)=:f(-q^4,-q^9), M(q)=:f(-q^5,-q^8), N(q)=:f(-q^6,-q^7).$$

Again, Berndt [4, p. 39, Entry 24(II)], we have

$$f_1^3=\sum_{n=0}^{\infty}(-1)^n(2n+1)q^{n(n+1)/2},$$

and its follows that

$$f_1^3 = A_0(q^{13}) - 3qA_1(q^{13}) + 5q^3A_2(q^{13}) - 7q^6A_3(q^{13}) + 9q^{10}A_4(q^{13}) - 11q^{15}A_5(q^{13}) + 13q^{21}A_6(q^{13}), \tag{34}$$

where $A_0, A_1, A_2, A_3, A_4, A_5$ and A_6 are series involving integral powers of q^{13}.
The following p-dissection identity holds.
From [6, Theorem 2.2], for any prime $p \geq 5$ and

$$\frac{\pm p-1}{6} = \begin{cases} \frac{p-1}{6}, & \text{if } p \equiv 1 \pmod 6 \\ \frac{-p-1}{6}, & \text{if } p \equiv -1 \pmod 6, \end{cases}$$

we have

$$f_1 = \sum_{\substack{k=-\frac{p-1}{2} \\ k\neq\frac{\pm p-1}{6}}}^{\frac{p-1}{2}} (-1)^k q^{\frac{3k^2+k}{2}} f\left(-q^{\frac{3p^2+(6k+1)}{2}}, -q^{\frac{3p^2-(6k+1)}{2}}\right) + (-1)^{\frac{\pm p-1}{6}} q^{\frac{p^2-1}{24}} f_{p^2}. \tag{35}$$

Furthermore, if $-\frac{p-1}{2} \leq k \leq \frac{p-1}{2}$ and $k \neq \frac{\pm p-1}{6}$, then $\frac{3k^2+k}{2} \not\equiv \frac{p^2-1}{24} \pmod p$.
From [2, Lemma 2.3], for any prime $p \geq 3$, we have

$$f_1^3 = \sum_{\substack{k=0 \\ k\neq\frac{p-1}{2}}}^{p-1} (-1)^k q^{\frac{k^2+k}{2}} \sum_{n=0}^{\infty} (-1)^n (2pn+2k+1) q^{pn\cdot\frac{pn+2k+1}{2}} + p(-1)^{\frac{p-1}{2}} q^{\frac{p^2-1}{3}} f_{p^2}^3. \tag{36}$$

Furthermore, if $0 \leq k \leq p-1$ and $k \neq \frac{p-1}{2}$, then $\frac{k^2+k}{2} \not\equiv \frac{p^2-1}{8} \pmod p$.
From Hirschhorn [8], we note that, if

$$\eta := \frac{f_1}{q^7 f_{169}} \text{ and } S := \frac{f_{13}^2}{q^{13} f_{169}^2}, \tag{37}$$

then

$$
\begin{aligned}
H_{13}(\eta) &= 1,\\
H_{13}(\eta^2) &= -2S-1,\\
H_{13}(\eta^3) &= 13,\\
H_{13}(\eta^4) &= 2S^2-13,\\
H_{13}(\eta^5) &= -20S^2-10\times 13S-13^2,\\
H_{13}(\eta^6) &= 10S3-13^2,\\
H_{13}(\eta^7) &= 98S^3+28\times 13S^2-13^3,\\
H_{13}(\eta^8) &= -70S^4-13^3,\\
H_{13}(\eta^9) &= -162S^4+108\times 13S^3+72\times 13^2S^2+18\times 13^3S+13^4,\\
H_{13}(\eta^{10}) &= 238S^5-13^4,\\
H_{13}(\eta^{11}) &= -902S^5-1672\times 13S^4-792\times 13^2S^3-198\times 13^3S^2\\
&\quad -22\times 13^4S-13^5,\\
H_{13}(\eta^{12}) &= -418S^6-13^5,\\
H_{13}(\eta^{13}) &= S^7+51\times 13^2S^6+808\times 13^2S^5+398\times 13^3S^4\\
&\quad +110\times 13^4S^3+16\times 13^5S^2+13^6S,\\
H_{13}(\eta^{14}) &= -506S^7-13^6. \qquad (38)
\end{aligned}
$$

From binomial theorem for any prime p, we have

$$f_1^p \equiv f_p \pmod{p}. \tag{39}$$

$$f_1^{p^2} \equiv f_p^p \pmod{p^2}. \tag{40}$$

$$pf_1^p \equiv pf_p \pmod{p^2}. \tag{41}$$

3. Proofs of Theorems 1.1 -1.4

***Proof of Theorem* 1.1.** Setting $r=13k+10$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{13k+10}(n)q^n = \frac{1}{f_1^{13k+10}}.$$

Using (39) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{13k+10}(n)q^n \equiv \frac{f_1^3}{f_{13}^{k+1}} \pmod{13}. \tag{42}$$

Using identity (34) in identity (42) and drawing out the common terms of $q^{13n+\nu}$ which occurs on both sides where $\nu \in \{4,5,7,8,9,11,12\}$, we complete the proof of identity (2).

Setting $r=13k+12$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{13k+12}(n)q^n = \frac{1}{f_1^{13k+12}}.$$

Using (39) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{13k+12}(n)q^n \equiv \frac{f_1}{f_{13}^{k+1}} \pmod{13}. \tag{43}$$

On employing identity (33) in identity(43) and drawing out the common terms of $q^{13n+\mu}$ which occurs on both sides, where $\mu \in \{3,4,6,8,10,11\}$, we complete the proof of identity (3). □

***Proof of Theorem* 1.2.** Setting $r = 169k + 8$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{169k+8}(n)q^n = \frac{1}{f_1^{169k+8}}.$$

Using (39) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{169k+8}(n)q^n \equiv \frac{f_1^5}{f_{169}^k f_{13}} \pmod{13}.$$

Employing (37) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{169k+8}(n)q^n \equiv \frac{q^{35}\eta^5}{f_{169}^{k-5} f_{13}} \pmod{13}. \tag{44}$$

Drawing out common terms of q^{13n+9} and applying operator H_{13} in (44), we obtain

$$\sum_{n=0}^{\infty} p_{169k+8}(13n+9)q^{13n+9} \equiv \frac{q^{35}H_{13}(\eta^5)}{f_{169}^{k-5} f_{13}} \pmod{13}. \tag{45}$$

From identities (38) and (45), we have

$$\sum_{n=0}^{\infty} p_{169k+8}(13n+9)q^{13n+9} \equiv 6q^9 \frac{f_{13}^3}{f_{169}^{k-1}} \pmod{13}.$$

Dividing the resulting identity by q^9 and changing q to $q^{1/13}$, we obtain

$$\sum_{n=0}^{\infty} p_{169k+8}(13n+9)q^n \equiv 6\frac{f_1^3}{f_{13}^{k-1}} \pmod{13}. \tag{46}$$

Applying identity (34) in identity (46) and drawing out the common terms of $q^{13n+\nu}$ which occurs on both sides where $\nu \in \{4,5,7,8,9,11,12\}$, we complete the proof of identity (4).
We omit the proofs of identities (5) to (13), since their proofs are similar to proof of identity (4). □

***Proof of Theorem* 1.3.** Setting $r = 169k + 34$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{169k+34}(n)q^n = \frac{1}{f_1^{169k+34}}.$$

Using (39) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{169k+34}(n)q^n \equiv \frac{f_1^5}{f_{169}^k f_{13}^3} \pmod{13}.$$

Employing (37) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{169k+34}(n)q^n \equiv \frac{q^{35}\eta^5}{f_{169}^{k-5}f_{13}^3} \pmod{13}. \tag{47}$$

Drawing out common terms of q^{13n+9} and applying operator H_{13} in (47), we obtain

$$\sum_{n=0}^{\infty} p_{169k+34}(13n+9)q^{13n+9} \equiv \frac{q^{35}H_{13}(\eta^5)}{f_{169}^{k-5}f_{13}^3} \pmod{13}, \tag{48}$$

From identities(38) and (48), we have

$$\sum_{n=0}^{\infty} p_{169k+34}(13n+9)q^{13n+9} \equiv 6q^9\frac{f_{13}}{f_{169}^{k-1}} \pmod{13},$$

Dividing the resulting identity by q^9 and changing q to $q^{1/13}$, we obtain

$$\sum_{n=0}^{\infty} p_{169k+34}(13n+9)q^n \equiv 6\frac{f_1}{f_{13}^{k-1}} \pmod{13}. \tag{49}$$

Employing identity (34) in identity (49) and drawing out the common terms of $q^{13n+\mu}$ which occurs on both sides where $\mu \in \{3,4,6,8,10,11\}$, we complete the proof of identity (14).
We omit the proofs of identities (15) to (22), since their proofs are similar to proof of identity (14). □

***Proof of Theorem* 1.4.** Setting $r = 169k+47$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{169k+47}(n)q^n = \frac{1}{f_1^{169k+47}}.$$

Using (39) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{169k+47}(n)q^n \equiv \frac{f_1^5}{f_{169}^{k}f_{13}^4} \pmod{13}.$$

Employing (37) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{169k+47}(n)q^n \equiv \frac{q^{35}\eta^5}{f_{169}^{k-5}f_{13}^4} \pmod{13}. \tag{50}$$

Drawing out common terms of q^{13n+9} and applying operator H_{13} in (50), we obtain

$$\sum_{n=0}^{\infty} p_{169k+47}(13n+9)q^{13n+9} \equiv \frac{q^{35}H_{13}(\eta^5)}{f_{169}^{k-5}f_{13}^4} \pmod{13}, \tag{51}$$

From identities(38) and (51), we have

$$\sum_{n=0}^{\infty} p_{169k+47}(13n+9)q^{13n+9} \equiv 6q^9\frac{1}{f_{169}^{k-1}} \pmod{13}.$$

Dividing the resulting identity by q^9 and changing q to $q^{1/13}$, then from resulting identities drawing out common terms of $q^{13n+\kappa}$ which occurs on both sides where $\kappa \in \{1,2,3,4,5,6,7,8,9,10,11,12\}$, we complete the proof

of identity (23).
We omit the proofs of identities (24) to (30), since proofs are similar to proof of identity (23). □

4. Proofs of Theorems 1.5 and 1.6

Proof of Theorem **1.5.** Setting $r = p^2k + p^2 - 1$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-1}(n)q^n = \frac{1}{f_1^{p^2k+p^2-1}}.$$

Using (40) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-1}(n)q^n \equiv \frac{f_1}{f_p^{p(k+1)}} \pmod{p^2}. \tag{52}$$

Employing (35) in (52), we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-1}(n)q^n$$
$$\equiv \frac{1}{f_p^{p(k+1)}} \sum_{\substack{m=-\frac{p-1}{2} \\ m\neq \frac{\pm p-1}{6}}}^{\frac{p-1}{2}} (-1)^m q^{\frac{3m^2+m}{2}} f\left(-q^{\frac{3p^2+(6m+1)}{2}}, -q^{\frac{3p^2-(6m+1)}{2}}\right)$$
$$+\frac{1}{f_p^{p(k+1)}}(-1)^{\frac{\pm p-1}{6}} q^{\frac{p^2-1}{24}} f_{p^2} \pmod{p^2}. \tag{53}$$

For any prime p with $-\frac{p-1}{2} \le m \le \frac{p-1}{2}$, consider the following congruence

$$\frac{3m^2+m}{2} \equiv \frac{p^2-1}{24} \pmod{p},$$

which is equivalent to $(6m+1)^2 \equiv 0 \pmod{p}$, since it has only solution $m = \frac{\pm p-1}{6}$.
Drawing out common terms of $q^{pn+\frac{p^2-1}{24}}$ which occurs on both sides in (53) then dividing the resulting identity by $q^{(p^2-1)/24}$ and changing q to $q^{1/p}$, we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-1}\left(pn + \frac{p^2-1}{24}\right)q^n \equiv \frac{(-1)^{\frac{\pm p-1}{6}} f_p}{f_1^{p(k+1)}} \pmod{p^2}.$$

From above identity if $k+1$ is multiples of p, then we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-1}\left(pn + \frac{p^2-1}{24}\right)q^n \equiv \frac{(-1)^{\frac{\pm p-1}{6}} f_p}{f_p^{pm}} \pmod{p^2}. \tag{54}$$

Drawing out common terms of $q^{pn+\nu}$ which occurs on both sides in (54) where $\nu \in \{1, 2, ..., p-1\}$, we complete the proof of identity (31). □

***Proof of Theorem* 1.6.** Setting $r = p^2k + p^2 - 3$ in (1), we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-3}(n)q^n = \frac{1}{f_1^{p^2k+p^2-3}}.$$

Using (40) in above identity, we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-3}(n)q^n \equiv \frac{f_1^3}{f_p^{p(k+1)}} \pmod{p^2}. \tag{55}$$

Employing identity (36) in identity (55), we obtain

$$\begin{aligned} \sum_{n=0}^{\infty} p_{p^2k+p^2-3}(n)q^n \equiv & \frac{1}{f_p^{p(k+1)}} \sum_{\substack{m=0 \\ m\neq \frac{p-1}{2}}}^{p-1} (-1)^m q^{\frac{m^2+m}{2}} \\ & \sum_{n=0}^{\infty} (-1)^n (2pn+2m+1) q^{pn\cdot\frac{pn+2m+1}{2}} \\ & + \frac{1}{f_p^{p(k+1)}} \cdot p(-1)^{\frac{p-1}{2}} q^{\frac{p^2-1}{3}} f_{p^2}^3 \pmod{p^2}. \end{aligned} \tag{56}$$

For any prime p with $0 \le m \le p-1$, consider

$$\frac{m^2+m}{2} \equiv \frac{p^2-1}{8} \pmod{p},$$

The above congruence is equivalent to $(2m+1)^2 \equiv 0 \pmod{p}$, since it has only solution $m = \frac{p-1}{2}$.

Drawing out the common terms of $q^{pn+\frac{p^2-1}{8}}$ which occurs on both sides in (56) then dividing the resulting identity by $q^{(p^2-1)/8}$ and changing q to $q^{1/p}$, we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-3}\left(pn + \frac{p^2-1}{8}\right) q^n \equiv \frac{(-1)^{\frac{p-1}{2}} p f_p}{f_1^{p(k+1)}} \pmod{p^2}. \tag{57}$$

Employing identity (41) in identity (57), we obtain

$$\sum_{n=0}^{\infty} p_{p^2k+p^2-1}\left(pn + \frac{p^2-1}{8}\right) q^n \equiv \frac{(-1)^{\frac{p-1}{2}} p}{f_p^k} \pmod{p^2}.$$

Drawing out the common terms of $q^{pn+\nu}$ which occurs on both sides where $\nu \in \{1, 2, ..., p-1\}$ in above identity, we complete the proof of identity (32). □

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